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Regge trajectories for mesons in the holographic dual of large-N_c QCD

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Abstract: We discuss Regge trajectories of dynamical mesons in large-N_c QCD, using the supergravity background describing N_c D4-branes compactified on a thermal circle. The flavor degrees of freedom arise from the addition of $N_f \ll N_c$ D6 probe branes. Our work provides a string theoretical derivation, via the gauge/string correspondence, of a phenomenological model describing the meson as rotating point-like massive particles connected by a flux string. The massive endpoints induce nonlinearities for the Regge trajectory. For light quarks the Regge trajectories of mesons are essentially linear. For massive quarks our trajectories qualitatively capture the nonlinearity detected in lattice calculations.

Keywords: AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction and summary

The Regge trajectories of mesons were among the first indications that mesons admit a stringy behavior. Indeed, a very elementary treatment of a classical spinning open bosonic string in flat space time yields a simple relation between the angular momentum and energy, $J = \alpha' E^2$, which is a Regge trajectory. Quantum corrections of the classical bosonic string configuration add an intercept to the trajectory, namely $J = \alpha' E^2 + \frac{(d-2)\pi}{12} + O(E^{-2})$.

In the modern era of the gauge/gravity holographic duality one wonders whether a similar behavior characterizes the classical spinning string in supergravity backgrounds that are duals of confining gauge theories. In [1] folded closed spinning strings were analyzed in the context of the KS and MN backgrounds. It was found that classically these configurations indeed admit a linear Regge behavior $J = \alpha'_{\text{eff}} E^2$ where $\alpha'_{\text{eff}} = \frac{\alpha'}{g_{00}(\hat{U})}$ and where $\hat{U}$ denotes the location of the spinning string along the radial direction. Interestingly, it was found that the quantum corrected trajectory is different than that of the bosonic string in flat space time, it takes the form $J = \alpha'_{\text{eff}} E^2 + \alpha_0 + \beta E$ where α_0 is not a pure number but rather depends on the parameters of the background, and so is β. These configurations map into the Regge trajectories of glueballs in the dual gauge theory. Glueball Regge trajectories with nonvanishing intercept were also found in lattice YM calculations [2].

There are several models of supergravity backgrounds holographically dual to confining gauge theories. In this paper we will make use of the one associated with the near extremal N_c D4 branes [3]. By imposing anti-periodic boundary conditions along the thermal circle for the fermions, the background corresponds to the low energy regime of the pure YM theory in four dimensions. The stringy Wilson loop in this background was computed in [4] using the methods of [5]. Whereas the Regge behavior of glueballs can be addressed in this model [6], mesonic spinning strings cannot be described in this framework since its dual theory does not include quarks in the fundamental representations. Flavor fundamental quarks and anti-quarks can be invoked in supergravity backgrounds by introducing flavor

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probe branes [7]. Probe D7 branes were incorporated in models based on $D3$ branes in [8–11]. Similar consideration of flavor can be found in [12]. The idea behind some these papers has been to extract the mesonic spectrum from the fluctuations of the fields, like pseudo scalars and vectors, on the flavor probe branes. This, obviously, cannot be done in supergravity for states with spin higher than two. To describe the Regge trajectories associated with high spins, one is naturally forced to redo the "old" calculations of spinning strings, but now not in the context of the bosonic string in 4d but rather in the context of the supergravity confining backgrounds that include probe branes. Concretely we followed in this work the proposal of [10] of adding D6 probe branes to the background of the near extremal N_c D4 branes.

By analyzing the conditions of having non trivial spinning strings, we show that the conditions of having spinning strings at a constant radial coordinate, resembles the conditions of having an area law Wilson line [13] and that it can occur only along the " wall of space" in the radial direction.

An interesting result of our analysis is the fact that the solution of the spinning strings is very closely related to a toy model of a string with two masses attached to its endpoints, which spins in flat space-time. This simple model was extensively considered in the 70's by, for example, [14, 15] and it has been reviewed in the book [16] where further references can be found. More importantly, this model which is also referred to as the mass-loaded generalization of the Chew-Frautschi formula, is an essential tool in the extremely successful approach to meson and hadron spectroscopy developed recently by Wilczek [17].

We describe the toy model and solve it prior to performing the stringy calculations. The main outcome of this paper is that the classical spinning strings with endpoints on the probe branes, admit Regge trajectories which get corrections due the "masses of the quarks"

$$E = \frac{2T_g}{\omega}\left(\arcsin x + \frac{1}{x}\sqrt{1-x^2}\right), \qquad J = \frac{T_g}{\omega^2}\left(\arcsin x + \frac{3}{2}x\sqrt{1-x^2}\right), \tag{1.1}$$

where x is the speed of the endpoints of the strings and the mass of the quarks is $m_q = T_g(1-x^2)/(\omega x)$. The Regge regime requires relativistic motion of the string, that is, $x \to 1$. The massive endpoints induce nonlinearities for the Regge trajectory. For light quarks the Regge trajectories of mesons are essentially linear. For massive quarks our trajectories qualitatively capture the nonlinearity detected in lattice calculations. In particular, the slope for the lowest states within the Regge trajectories of heavy-quark mesons is flavor dependent, while for the highest states the slope is universal. We explain the universality of the second slope in terms of generic properties of confining holographic backgrounds.

The organization of the paper is as follows. In section 2 we describe the general setup of spinning strings in confining backgrounds. Section 3 is devoted to reviewing a toy model consisting of two massive relativistic particles attached to the ends of a spinning string. The energy and angular momentum of the system are computed and are shown to admit a Regge trajectory behavior with corrections that depend on mass of the particles. In section 4 we analyze the Regge trajectories of mesons from the perspective of the gauge/string duality. We investigate a macroscopic spinning open string configuration, whose endpoints are

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located at the boundary of the flavor brane. It is shown that one can approximate its configuration with that of two vertical strings stretched between the flavor brane and the "floor" and a horizontal string that stretches along the wall. This classical configuration has a Regge like trajectory which indeed can be mapped into that of the toy model. Finally, in section 5 we comment on the rapport between the toy model and the more recent literature on the phenomenology of mesons.

2. Spinning open strings in confining backgrounds: General setup

Consider a background metric of the form

$$ds^2 = -g_{00}(U)dt^2 + g_{ii}(U)(dx^i)^2 + g_{UU}dU^2 + \cdots\,, \tag{2.1}$$

where x^i are the space coordinates associated with the uncompactified worldvolume, U is a radial coordinates and ellipsis stands for additional transverse directions. The metric as well as any other field of the background depend only on the radial direction U.

In [13] it was shown that backgrounds with metric of this form admit an area law behavior, namely confinement, if one of the two conditions is obeyed

$$g_{00}g_{ii}(U) \quad \text{has a minimum at} \, quad U = U_{\min} \qquad \text{and} \qquad g_{00}g_{ii}(U_{\min}) > 0\,, \tag{2.2}$$

$$g_{00}g_{UU}|_{U=U_\Lambda} \to \infty \qquad \text{and} \qquad g_{00}g_{ii}(U_\Lambda) > 0\,. \tag{2.3}$$

Backgrounds that obey the first condition are for instance the KS and the MN backgrounds and their non-supersymmetric deformations. For the metric of the $AdS_5 \times S^5$ black hole and its non extremal D_p analogs the second condition is obeyed. In both cases the string that associates with the Wilson line stretches vertically from the boundary to the "wall"(or "end of space" or "floor") where it stretches horizontally and then up vertically to the boundary. In the horizontal segment the string is in fact that of a flat space time and hence the area law behavior. We will see belove that the condition of having a spinning string with a constant radial coordinate will be similar to those of having confining Wilson loop, and the string will spin along the "wall".

Since we have in mind addressing spinning strings, it is convenient to describe the space part of the metric as

$$dx_i^2 = dR^2 + R^2 d\theta^2 + dx_3^2\,, \tag{2.4}$$

where x_3 is the direction perpendicular to the plan of rotation. The classical equations of motion of a bosonic string defined on this background can be formulated on equal footing in the NG formulation or the Polyakov action. Let us use now the latter. The equations of motion associated with the variation of t, θ, R and U respectively are

$$\begin{aligned}
\partial_\alpha(g_{00}\partial^\alpha t) &= 0 \\
\partial_\alpha(g_{ii}R^2\partial^\alpha \theta) &= 0 \\
\partial_\alpha(g_{ii}\partial^\alpha R) - g_{ii}R\partial_\alpha\theta\partial^\alpha\theta &= 0 \\
2\partial_\alpha(g_{UU}\partial^\alpha U) + \frac{dg_{00}}{dU}\partial_\alpha t\partial^\alpha t - \frac{dg_{ii}}{dU}\partial_\alpha x^i\partial^\alpha x^i - \frac{dg_{UU}}{dU}\partial_\alpha u\partial^\alpha u &= 0\,,
\end{aligned} \tag{2.5}$$

where α denotes the worldsheet coordinates τ and σ. In addition in the Polyakov formulation one has to add the Virasoro constraint

$$-g_{00}(\partial_\pm t)^2 + g_{ii}(\partial_\pm x^i)^2 + g_{UU}(\partial_\pm U)^2 + \cdots = 0\,, \tag{2.6}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ and ... stands for the contribution to the Virasoro constraint of the rest of the background metric.

Next we would like to find solutions of the equations of motion which describe strings spinning in space-time. For that purpose we take the following ansatz

$$t = \tau \qquad \theta = \omega\tau \qquad R(\sigma\tau) = R(\sigma) \qquad U = \hat{U} = \text{constant}\,. \tag{2.7}$$

It is obvious that this ansatz solves the first two equations. The third equation together with the Virasoro constraint is solved (for the case that $g_{00} = g_{ii}$) by $R = Acos(\omega\sigma) + Bsin(\omega\sigma)$ with $\omega^2(A^2+B^2) = 1$. The boundary conditions we want to impose will select the particular combination of A and B. Let us now investigate the equation of motion associated with U and for the particular ansatz $U = \hat{U}$. This can be a solution only provided

$$\frac{dg_{00}}{dU}|_{U=\hat{U}} = 0 \qquad \frac{dg_{ii}}{dU}|_{U=\hat{U}} = 0\,. \tag{2.8}$$

This is just the first condition for having a confining background. The condition $g_{00}g_{ii}(\hat{U}) > 0$ insures that the Virasoro constraint is obeyed in a non trivial manner. In [1] spinning strings in the KS and MN models, which belong to this class of confining backgrounds, were analyzed.

In the background that we will employ in the present paper, the near extremal $D4$ brane, one direction is along an S^1 and there is no five dimensional Lorentz invariance. The direction along the S^1 denoted by ψ has a metric which is related to that of the U direction as follows $g_{\psi\psi} = [g_{UU}]^{-1}$. Thus the condition of having a solution of the equation of motion with $U = \hat{U}$ includes the condition

$$\frac{dg_{\psi\psi}}{dU}|_{U=\hat{U}} = -\frac{\frac{dg_{UU}}{dU}}{g_{UU}^2}|_{U=\hat{U}} = 0\,. \tag{2.9}$$

This condition is obeyed if at $U = \hat{U}$ $g_{UU}(\hat{U}) \to \infty$ which is the second condition for a confining background (2.2) with $\hat{U} = U_\Lambda$, and again with the the demand of non-vanishing $g_{00}g_{ii}(\hat{U}) > 0$ to have a non-trivial Virasoro constraint. The left over part of the equation of motion (2.8) will be shown to be obeyed in section 4 by transforming the radial coordinate to a coordinate that measures the distance from U_Λ.

To summarize, we have just realized that there is a close relation between the conditions of having area law Wilson loop and of having a spinning string configuration at a constant radial coordinate.

3. A toy model of the meson

Prior to analyzing the string configuration that describes a meson, let first review a toy model that consists of of two massive, relativistic particles connected by a string (see figure 1).

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Such models were proposed in the past as an effective description of mesons [15, 16, 18]. However, the reason we address them here is that they are intimately related to the stringy meson associated with a confining background, as we will show in the next section.

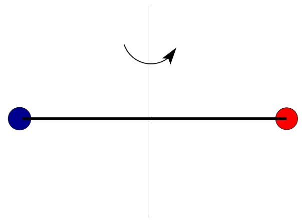

Figure 1: The toy model.

Consider the flat space-time NG action of an open string combined with the action of two relativistic particles of equal mass m attached to the endpoints of the string.

$$
\begin{aligned}
S = &-T\int d\tau \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\sigma \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2][(t')^2 - R'^2 - R^2\theta'^2] + [-\dot{t}t' + \dot{R}R' + R^2\dot{\theta}\theta']^2} - \\
&- m\int d\tau \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2]}|_{\sigma=-\frac{\pi}{2}} - m\int d\tau \sqrt{-[(\dot{t})^2 - \dot{R}^2 - R^2\dot{\theta}^2]}|_{\sigma=\frac{\pi}{2}}\,, \qquad (3.1)
\end{aligned}
$$

where the metric was taken to be $ds^2 = -dt^2 + dR^2 + R^2 d\theta^2$ is the space-time metric relevant to the rotating string, T is the effective string tension m is the mass of the particles, the world volume coordinate σ is defined in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\dot{A}$ and A' denote derivatives with respect to τ and σ respectively.

Note that in the string part of the action X^0, R and θ are *a priori* functions of σ and τ and in the particle only of τ. Moreover, due to the reflection symmetry of the problem, we take that the values of t, R and θ at σ equal those of t, R and θ at $-\sigma$. Next we discuss the equations of motion. Due to the fact that there are particle actions at the end points of the interval of σ, the equations of motion of x^μ will include a bulk equation and a surface one as can be see from the following variation

$$
\begin{aligned}
\delta S = &-T\int d\tau \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\sigma \left[\frac{\partial \mathcal{L}_s}{\partial X^\mu} - \partial_\alpha\left[\frac{\partial \mathcal{L}_s}{\partial_\alpha X^\mu}\right]\right]\delta X^\mu(\sigma,\tau) + \\
&+ \int d\tau \left[T\frac{\partial \mathcal{L}_s}{\partial_\sigma X^\mu}\delta X^\mu(\sigma,\tau)|_{\sigma=-\frac{\pi}{2}} - m\left[\partial_\tau\left[\frac{\partial \mathcal{L}_p}{\partial_\tau X^\mu}\right] - \frac{\partial \mathcal{L}_p}{\partial X^\mu}\right]\delta X^\mu(\tau)\right] - \\
&- \int d\tau \left[T\frac{\partial \mathcal{L}_s}{\partial_\sigma X^\mu}\delta X^\mu(\sigma,\tau)|_{\sigma=\frac{\pi}{2}} - m\left[\partial_\tau\left[\frac{\partial \mathcal{L}_p}{\partial_\tau X^\mu}\right] - \frac{\partial \mathcal{L}_p}{\partial X^\mu}\right]\delta X^\mu(\tau)\right]. \qquad (3.2)
\end{aligned}
$$

Thus the equations of motion are

$$
\begin{aligned}
\frac{\partial \mathcal{L}_s}{\partial X^\mu} - \partial_\alpha\left[\frac{\partial \mathcal{L}_s}{\partial_\alpha X^\mu}\right] &= 0\,, \\
T\frac{\partial \mathcal{L}_s}{\partial_\sigma X^\mu}|_{\sigma=-\frac{\pi}{2}} - m\partial_\tau\left[\frac{\partial \mathcal{L}_p}{\partial_\tau X^\mu}\right] + m\frac{\partial \mathcal{L}_p}{\partial X^\mu} &= 0\,, \\
T\frac{\partial \mathcal{L}_s}{\partial_\sigma X^\mu}|_{\sigma=\frac{\pi}{2}} - m\partial_\tau\left[\frac{\partial \mathcal{L}_p}{\partial_\tau X^\mu}\right] + m\frac{\partial \mathcal{L}_p}{\partial X^\mu} &= 0\,. \qquad (3.3)
\end{aligned}
$$

We now look for spinning configurations based on the following ansatz (2.7)

$$
t = \tau\,, \qquad \theta = \omega\tau\,, \qquad R(\tau,\sigma) = R(\sigma)\,. \qquad (3.4)
$$

For this type of ansatz the bulk equations of motion are

$$
\begin{aligned}
\partial_\tau\left(\frac{R'(\sigma)}{\sqrt{1-\omega^2R^2(\sigma)}}\right) &= 0\,,\\
\partial_\tau\left(\frac{\omega R'(\sigma)}{\sqrt{1-\omega^2R^2(\sigma)}}\right) &= 0\,,\\
\partial_\sigma\left(\sqrt{1-\omega^2R^2(\sigma)}\right)+\frac{\omega^2RR'(\sigma)}{\sqrt{1-\omega^2R^2(\sigma)}} &= 0\,.
\end{aligned}
\tag{3.5}
$$

The first and second equations are trivially obeyed. The third equation is obeyed in fact for any $R(\sigma)$. The particular form of it depends on the boundary conditions that one wants to impose. For instance, for the closed string, requiring the periodicity $R(\sigma) = R(\sigma + 2\pi)$, where the range of σ is taken to be $(0, 2\pi)$, is compatible with a solution of the form $R = \frac{1}{\omega}\cos(\sigma)$. Or, for the case of rotation Wilson line one imposes $R(0) = 0$ and $R(\pi) = L$ and the corresponding solution $R = Lsin(\sigma/4)$.

In the present case the boundary conditions will be determined by the surface equations of motions. In fact the surface equations for t and θ are trivial, and only the equations for R constrain the solutions. The latter reads

$$
T\left(\frac{\partial\mathcal{L}_s}{\partial R'}\right)\bigg|_{R=R_0}+m\left(\frac{\partial\mathcal{L}_p}{\partial R}\right)=0\,,
$$

which implies

$$
T\sqrt{1-\omega^2R_0^2}=m\frac{R_0\omega^2}{\sqrt{1-\omega^2R_0^2}}\,,
\tag{3.6}
$$

where $R_0 \equiv R(-\pi/2) = R(\pi/2)$. This expression enables us to determine ω in terms of R_0 or vice-versa. For the case of $m = 0$, namely without the action of the particles, the surface term of the string action should vanish. Note that the same relation can be derived by minimizing the action with respect to R_0.

Next we want to find the energy and the angular momentum of the system. Starting with the action 3.1 the energy and angular momentum are given by

$$
\begin{aligned}
E &= T\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}d\sigma\frac{\dot t\sqrt{[(t')^2-R'^2-R^2\theta'^2]}}{\sqrt{-[(\dot t)^2-\dot R^2-R^2\dot\theta^2]}}+m\frac{\dot t}{\sqrt{-[(\dot t)^2-\dot R^2-R^2\dot\theta^2]}}|_{\sigma=-\pi/2,\pi/2}\\
&= 2T\int_0^{R_0}dR\frac{1}{\sqrt{1-\omega^2R^2(\sigma)}}+2m\frac{1}{\sqrt{1-\omega^2R_0^2}}=2\frac{T}{\omega}\arcsin(\omega R_0)+2m\frac{1}{\sqrt{1-\omega^2R_0^2}}\\
J &= T\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}d\sigma\frac{R^2\dot\theta\sqrt{[(t')^2-R'^2-R^2\theta'^2]}}{\sqrt{-[(\dot t)^2-\dot R^2-R^2\dot\theta^2]}}+m\frac{R_0^2\dot\theta}{\sqrt{-[(\dot t)^2-\dot R^2-R^2\dot\theta^2]}}|_{\sigma=-\pi/2,\pi/2}\\
&= 2T\omega\int_0^{R_0}dR\frac{R^2}{\sqrt{1-\omega^2R^2(\sigma)}}+2m\frac{R_0^2\omega}{\sqrt{1-\omega^2R_0^2}}\\
&= \frac{T}{\omega^2}\left[\arcsin(\omega R_0)-\omega R_0\sqrt{1-\omega^2R^2(\sigma)}\right]+2m\frac{\omega R_0^2}{\sqrt{1-\omega^2R_0^2}}\,.
\end{aligned}
$$

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So we can summarize the results as follows

$$E = 2\frac{T}{\omega}\left[\arcsin(\omega R_0) + 2\sqrt{\frac{m}{TR}}\right],$$
$$J = \frac{T}{\omega^2}\left[\arcsin(\omega R_0) + 3(\omega R_0)^2\sqrt{\frac{m}{TR_0}}\right]. \tag{3.7}$$

Obviously for $m = 0$ we are back to the standard linear Regge trajectory. It is straightforward to generalize the case of a string with equal masses at his endpoints to the case where at each endpoint there are two different masses. In this case we get two "sewing" conditions instead of (3.6) and there will be contributions to the energy and J from the two masses expressed in terms of two separation distances.

In the next section the relation between this toy model and the stringy meson will be made clear by deriving a similar relation between J and E.

4. The meson as a rotating string

In this section we derive the Regge trajectory of a meson with large spin, from the perspective of the gauge/string duality. For specificity the background we consider is the supergravity background of a large number N_c of D4 branes compactified on a thermal circle. By adding $N_f \ll N_c$ D6 probe branes (by probe we mean that we ignore the back-reaction of the D6 branes), this geometry becomes the holographic dual of 4 dimensional large-N_c QCD, with N_f flavors plus KK modes. Without loss of generality we restrict in the following to the case when $N_f = 1$. A meson with large spin will correspond to an open spinning string string in the supergravity background sourced by the the non-extremal D4 branes in the decoupling limit

$$ds^2 = \frac{U^{3/2}}{R_{D4}3/2}(dX^\mu dX^\nu \eta_{\mu\nu} + f(U)d\psi^2) + K(U)(d\rho^2 + \rho^2 d\Omega_4^2), \tag{4.1}$$

where

$$f(U) = 1 - \frac{{U_\Lambda}^3}{U^3},$$
$$U(\rho) = \left(\rho^{3/2} + \frac{U_\Lambda^3}{4\rho^{3/2}}\right)^{2/3},$$
$$K(U) = {R_{D4}}^{3/2}U^{1/2}\rho^{-2}. \tag{4.2}$$

The coordinate ψ parametrizes the thermal circle on which the D4 branes are compactified, and X^μ with $\mu = 0, 1, 2, 3$ are coordinates in the remaining 4 non-compact directions along the D4 branes. The endpoints of this macroscopic string are located on the D6 probe brane which extends in the ρ direction from the boundary $\rho = \infty$ to $\rho = \rho_f$.

More precisely, the D6 probe spans the same 4 non-compact directions X^μ as the D4 branes, plus 3 other non-compact directions in the transverse space to the D4, $\lambda^i, i = 1, 2, 3$:

$${ds_5}^2 = d\rho^2 + \rho^2 {d\Omega_4}^2 = d\lambda^i d\lambda^i + dr^2 + r^2 d\phi^2 = d\lambda^2 + \lambda^2 d\Omega_2^2 + dr^2 + r^2 d\phi^2. \tag{4.3}$$

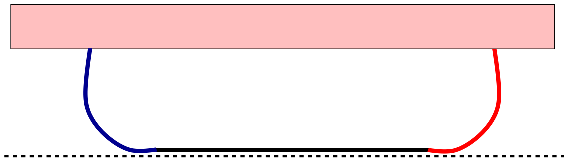

Figure 2: String ending on the probe D6.

Solving the equations of motion derived from the Born-Infeld action [10], the D6 probe profile $r = r(\lambda), \phi =$constant is described by the equation

$$\frac{d}{d\lambda}\left[\left(1+\frac{{U_\Lambda}^3}{4\rho^3}\right)^2\lambda^2\frac{dr/d\lambda}{\sqrt{1+(dr/d\lambda)^2}}\right] = -\frac{3}{2}\frac{{U_\Lambda}^3}{\rho^5}\left(1+\frac{{U_\Lambda}^3}{4\rho^3}\right)\lambda^2\frac{dr/d\lambda}{\sqrt{1+(dr/d\lambda)^2}}\,. \quad (4.4)$$

Due to the non-trivial profile of the D6 brane, the $U_A(1)$ symmetry is spontaneously broken and the quark condensate $\langle\bar{q}q\rangle$ acquires a vev. Asymptotically this symmetry is restored, $r = \text{constant} + \frac{c}{\lambda}$ and c is related to the quark condensate [10].

In terms of the extent of the D6 brane probe in the 5 dimensional space transverse to the D4 branes, we have $\rho^2_{D6} = r(\lambda)^2 + \lambda^2$. Thus, as advertised, the D6 stretches in the ρ directions from ρ_f at $\lambda = 0$[1] to ∞ at $\lambda \to \infty$.

We now return to the description of a meson of dynamical quarks as a rotating open string whose endpoints lie on the D6 probe. We begin with the ansatz

$$X^0 = e\tau\,, \qquad \theta = e\omega\tau\,, \qquad R = R(\sigma)\,, \qquad r = r(\sigma)\,, \qquad \lambda = \lambda(\sigma)\,, \quad (4.5)$$

where we parametrized the 4d non-compact space on the worldvolume of the non-extremal D4 branes by

$$dX^\mu dX_\mu = -(dX^0)^2 + dR^2 + R^2 d\theta^2 + (dX^3)^2\,. \quad (4.6)$$

The boundary conditions corresponding to this configuration are Neumann in the directions parallel with the D6 brane, and Dirichlet in the directions transverse to the brane:

$$\begin{aligned}\left(\partial_\sigma X^0 = \partial_\sigma\theta = \partial_\sigma R = \partial_\sigma\lambda\right)\Big|_{\sigma=0,\pi} &= 0\,,\\ r(\sigma) = r(\lambda(\sigma))\Big|_{\sigma=-\pi/2,\pi/2} &.\end{aligned} \quad (4.7)$$

Notice that the boundary conditions are trivially satisfied in the X^0 and θ coordinates. Plugging this ansatz in the NG action we search for a solution characterized by $\lambda =$ constant. The equation of motion associated with λ

$$\partial_\sigma\left(\frac{K\sqrt{(U/R_{D4})^{3/2}(\partial_\tau X^{02} - R^2\partial_\tau\theta^2)}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2 + K(\partial_\sigma r^2 + \partial_\sigma\lambda^2)}}\partial_\sigma\lambda\right) =$$
$$= \frac{\lambda}{\rho}\frac{d}{d\rho}\left(\sqrt{\left(\frac{U}{R_{D4}}\right)^{3/2}(\partial_\tau X^{02} - R^2\partial_\tau\theta^2)\left(\left(\frac{U}{R_{D4}}\right)^{3/2}\partial_\sigma R^2 + K(\partial_\sigma r^2 + \partial_\sigma\lambda^2)\right)}\right),$$

[1]It is easy to see that $d\rho/d\lambda = 2(\lambda + r(dr/d\lambda)) = 0$ at $\lambda = 0$.

is trivially satisfied for $\lambda = 0$. The picture that transpires out of this analysis is that of a spinning open string whose endpoints are located at the termination point of the D6 brane in the ρ direction.

Adding $\lambda = 0$ to the ansatz (4.5), the NG action reads:

$$S = -T_s \int d\sigma d\tau \sqrt{\left(\frac{U}{R_{D4}}\right)^{3/2} (\partial_\tau X^{02} - R^2 \partial_\tau \theta^2) \left(\left(\frac{U}{R_{D4}}\right)^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2\right)}, \tag{4.8}$$

where we used that for $\lambda = 0$ we have $r = \rho_{D6}$. From the variational principle

$$\begin{aligned}
0 \;=&\; \delta S \\
=&\; \int d\sigma d\tau \delta R \Bigg[-\partial_\sigma \left(\frac{(U/R_{D4})^{3/2} \partial_\sigma R \sqrt{(\partial_\tau X^{02} - R^2 \partial_\tau \theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2}} \right) - \\
&\quad - \frac{R \partial_\tau \theta^2 \sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2)}}{\sqrt{\partial_\tau X^{02} + R^2 \partial_\tau \theta^2}} \Bigg] + \\
&+ \int d\sigma d\tau \delta\rho \Bigg[-\partial_\sigma \left(\frac{K \partial_\sigma \rho \sqrt{(\partial_\tau X^{02} - R^2 \partial_\tau \theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2}} \right) + \\
&\quad + \frac{\sqrt{\partial_\tau X^{02} - R^2 \partial_\tau \theta^2} (3 \frac{dU}{d\rho} R_{D4}^{-1} ((U/R_{D4})^2 \partial_\sigma R^2 + 1/2 (U/R_{D4})^{1/2} K \partial_\sigma \rho^2) + (U/R_{D4})^{3/2} \frac{dK}{d\rho} \partial_\sigma \rho^2)}{2 \sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2)}} \Bigg] - \\
&- \int d\sigma d\tau \; \delta X^0 \partial_\tau \left(\frac{\partial_\tau X^0 \sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2)}}{\sqrt{\partial_\tau X^{02} - R^2 \partial_\tau \theta^2}} \right) + \\
&+ \int d\sigma d\tau \; \delta\theta \partial_\tau \left(\frac{R^2 \partial_\tau \theta \sqrt{(/R_{D4}) U^{3/2}((U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2)}}{\sqrt{\partial_\tau X^{02} - R^2 \partial_\tau \theta^2}} \right) + \\
&+ \int d\tau \delta R \frac{(U/R_{D4})^{3/2} \partial_\sigma R \sqrt{(\partial_\tau X^{02} - R^2 \partial_\tau \theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2}} \Bigg|_{\sigma=-\pi/2}^{\sigma=\pi/2} + \\
&+ \int d\tau \delta\rho \frac{K \partial_\sigma \rho \sqrt{(\partial_\tau X^{02} - R^2 \partial_\tau \theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2} \partial_\sigma R^2 + K \partial_\sigma \rho^2}} \Bigg|_{\sigma=-\pi/2}^{\sigma=\pi/2},
\end{aligned} \tag{4.9}$$

we extract the equations of motion and boundary conditions. As we have already mentioned, we impose Dirichlet boundary conditions in ρ and Neumann boundary conditions in R. Combining these two conditions into a single one we find that the string ends transversely to the probe brane, i.e. $d\rho/dR = d\rho/d\sigma \cdot d\sigma/dR = \infty$ at the endpoints of the string.

The equation of motion derived from the NG action is

$$\frac{d}{dR} \left[\frac{U \mathcal{E} \dot{\rho}}{\rho^2 \sqrt{U R_{D4}^{-3} + \frac{\dot{\rho}^2}{\rho^2}}} \right] = \frac{dU}{d\rho} \frac{\mathcal{E}}{\sqrt{U R_{D4}^{-3} + \frac{\dot{\rho}^2}{\rho^2}}} \left(\frac{3}{2} U R_{D4}^{-3} + \frac{\dot{\rho}^2}{\rho^2} \right) - \frac{\dot{\rho}^2 U \mathcal{E}}{\rho^3 \sqrt{U R_{D4}^{-3} + \frac{\dot{\rho}^2}{\rho^2}}}, \tag{4.10}$$

where we have introduced the notation

$$\mathcal{E} = \sqrt{e^2 - (e\omega)^2 R^2}, \tag{4.11}$$

and dots represent R-derivatives.

Given that the NG action (4.8) is invariant under shifts is X^0 and θ, we extract the associated conserved charges, namely the energy

$$E = T_s \int dR \frac{e\sqrt{(U/R_{D4})^{3/2} + K\dot{\rho}^2}}{\mathcal{E}} \left(\frac{U}{R_{D4}}\right)^{3/4}, \tag{4.12}$$

and the angular momentum

$$J = T_s \int dR \frac{e\omega R^2\sqrt{(U/R_{D4})^{3/2} + K\dot{\rho}^2}}{\mathcal{E}} \left(\frac{U}{R_{D4}}\right)^{3/4}. \tag{4.13}$$

We distinguish two region of interest spanned by the open string:

- Region I, characterized by $\dot{\rho} \to \infty$, and
- Region II, characterized by $\dot{\rho} \to 0$.

In the limit where the separation between the endpoints of the string is large, the vertical Region I and the horizontal Region II represent a good approximation to the shape of the string. However, we should point out that the separation distance is not a fixed parameter, rather it is a dynamical variable, and we will soon relate it to the external parameters of the problem: ω and ρ_f.

Region I can be viewed as extending from ρ_f, the location of the brane probe, to ρ_Λ, which is the end of space, at a fixed value of R which we denote by $\pm R_0$. Region II is the horizontal piece of the string, with the string extending from $-R_0$ to R_0 at fixed ρ.

Given that we introduced an additional quantity, namely the separation between the endpoints $2R_0$, and we decomposed the profile of the string into two regions, we must re-investigate the way the variational principle is satisfied. In (4.9) we notice that there is a surviving boundary term from Region II and a bulk term from Region I

$$\begin{aligned} &\int d\tau \delta R \frac{(U/R_{D4})^{3/2}\partial_\sigma R\sqrt{(\partial_\tau X^{02} - R^2\partial_\tau\theta^2)(U/R_{D4})^{3/2}}}{\sqrt{(U/R_{D4})^{3/2}\partial_\sigma R^2 + K\partial_\sigma\rho^2}}\Bigg|_{\sigma=-\alpha}^{\sigma=\alpha} - \qquad (4.14) \\ &-\left(\int_{\sigma=-\pi/2}^{\sigma=-\alpha} + \int_{\sigma=\alpha}^{\sigma=\pi/2}\right) d\sigma d\tau \delta R \frac{R\partial_\tau\theta^2\sqrt{(U/R_{D4})^{3/2}((U/R_{D4})^{3/2}\partial_\sigma R^2 + K\partial_\sigma\rho^2)}}{\sqrt{\partial_\tau X^{02} - R^2\partial_\tau\theta^2}}, \end{aligned}$$

where Region II corresponds to $\sigma \in (-\alpha, \alpha)$ and Region I to $\sigma \in (-\pi/2, -\alpha), \sigma \in (\alpha, \pi/2)$. Substituting the solution to the equations of motion into these terms we end up with

$$\int d\tau \delta R \sqrt{1 - \omega^2 R_0^2}(U_\Lambda/R_{D4})^{3/2} - \frac{\omega^2 R_0}{\sqrt{1-\omega^2 R_0^2}} \int d\tau \int_{\rho_\Lambda}^{\rho_f} d\rho \delta R \frac{U(\rho)}{\rho}. \tag{4.15}$$

The only way we can achieve cancellation is by requiring $\delta R(\rho, \tau) = \delta R(\tau)$ for $\rho \in (\rho_\Lambda, \rho_f)$ and by enforcing

$$1 - \omega^2 R_0^2 = \omega^2 R_0 \frac{1}{(U_\Lambda/R_{D4})^{3/2}} \int_{\rho_\Lambda}^{\rho_f} d\rho \frac{U(\rho)}{\rho}. \tag{4.16}$$

By recognizing on the rhs the mass of the dynamical quarks

$$m_q = T_s \int_{\rho_f}^{\rho_\Lambda} d\rho \sqrt{g_{00} g_{\rho\rho}} = T_s \int_{\rho_f}^{\rho_\Lambda} d\rho \frac{U}{\rho} \tag{4.17}$$

we found the desired relation between the separation distance, angular velocity and the mass of the quarks m_q (which is determined as a function of the position of the flavor brane ρ_f). As expected, having the string endpoints moving at the speed of light requires $m_q = 0$.

Let us analyze in detail the equations of motion.

- Region I: the equation of motion is satisfied to leading order, as we can see from

$$\frac{d}{dR}\left(\frac{U\mathcal{E}}{\rho}\right) = \left(\frac{dU}{d\rho} - \frac{U}{\rho}\right)\mathcal{E}\frac{\dot{\rho}}{\rho}\,. \tag{4.18}$$

  Substituting (4.2) it is easy to check that the lhs becomes precisely equal to the r.h.s.

  Assuming an almost rectangular shape, this region can be extended all the way to the end of space where the string flattens. In terms of ρ the end of space is marked by $\rho_\Lambda = (\frac{1}{2})^{2/3}U_\Lambda$.

  The contributions to the energy and angular momentum are

$$E_I = T_s \int_{\rho_\Lambda}^{\rho_f} d\rho \frac{e\sqrt{U^{3/2}(dR/d\rho)^2 + K}}{\mathcal{E}}(U/R_{D4})^{3/4} = \frac{eT_s}{\mathcal{E}}\Bigg|_{R=-R_0,R_0} \int d\rho\frac{U}{\rho} \tag{4.19}$$

$$J_I = T_s\frac{e\omega R^2}{\mathcal{E}}\Bigg|_{R=-L/2,L/2} \int d\rho\frac{U}{\rho}\,. \tag{4.20}$$

  Using that that the mass of the dynamical quarks is given by (4.17), we find the the energy and angular momentum contribution coming from the vertical regions of the Wilson loop reproduce the energy and angular momentum of relativistic spinning particles:

$$E_I = \frac{2m_q}{\sqrt{1-\omega^2 R_0^2}}\,, \qquad J_I = \frac{2m_q\omega {R_0}^2}{\sqrt{1-\omega^2R_0^2}}\,. \tag{4.21}$$

- Region II: as mentioned before, in this region the string is almost flat in the ρ direction. We notice that this can happen only for $\rho = \rho_\Lambda$, since from the equation of motion we find

$$\frac{d}{dR}\left(\frac{\sqrt{U}\mathcal{E}\dot{\rho}}{\rho^2}\right) = \frac{3}{2}\sqrt{U}\frac{dU}{d\rho}R_{D4}^{-3}\,\mathcal{E}. \tag{4.22}$$

  In the limit we are interested ($\dot{\rho} \to 0$), this becomes

$$\frac{\ddot{\rho}}{\rho^2}\mathcal{E} + \frac{\dot{\mathcal{E}}\dot{\rho}}{\rho^2} = \frac{3}{2}\frac{dU}{d\rho}R_{D4}^{-3}\,\mathcal{E}. \tag{4.23}$$

  The l.h.s. vanishes for constant ρ so we require that the rhs vanishes too. This is indeed the case for $\rho = \rho_\Lambda$, because $dU/d\rho|_{\rho_\Lambda} = 0$.

  The energy and angular momentum contributions of this region

$$E_{II} = T_s\int_{-R_0}^{R_0} dR\frac{e\sqrt{U^{3/2}+K\dot{\rho}^2}}{\mathcal{E}}\left(\frac{U}{R_{D4}}\right)^{3/4}$$

$$
\begin{aligned}
&= T_s \left(\frac{U_\Lambda}{R_{D4}}\right)^{3/2} \frac{2}{\omega} \arcsin(\omega R_0) && (4.24) \\
J_{II} &= T_s \int_{-R_0}^{R_0} dR\, e\omega R^2 \frac{e\sqrt{U^{3/2}+K\dot{\rho}^2}}{\mathcal{E}} U^{3/4} \\
&= T_s \left(\frac{U_\Lambda}{R_{D4}}\right)^{3/2} \frac{1}{\omega^2} \left(\arcsin(\omega R_0) - \omega R_0 \sqrt{1-\omega^2 {R_0}^2}\right) && (4.25)
\end{aligned}
$$

are those of an open string spinning in flat space, but with the string tension rescaled by $(U_\Lambda/R_{D4})^{3/2}$ to the value of the gauge theory quark-antiquark flux tube string tension $T_g = T_s(U_\Lambda/R_{D4})^{3/2}$.

Gathering all the terms, the energy and angular momentum of our Wilson loop are

$$
\begin{aligned}
E &= T_g \frac{2}{\omega} \arcsin(\omega R_0) + \frac{2m_q}{\sqrt{1-\omega^2 R_0^2}} && (4.26) \\
J &= T_g \frac{1}{\omega^2} \left(\arcsin(\omega R_0) - \omega R_0 \sqrt{1-\omega^2 {R_0}^2}\right) + \frac{2m_q \omega {R_0}^2}{\sqrt{1-\omega^2 R_0^2}}\,. && (4.27)
\end{aligned}
$$

Making use of the "sewing" condition (4.16), which we can now rewrite as

$$
1-\omega^2 R_0^2 = \frac{\omega^2 R_0 m_q}{T_g}\,, \tag{4.28}
$$

in order to eliminate the dependence on one of the parameters ω and R_0, we find that the energy and angular momentum are given by:

$$
\begin{aligned}
E &= \frac{2T_g}{\omega}\left(\arcsin x + \frac{1}{x}\sqrt{1-x^2}\right), \\
J &= \frac{T_g}{\omega^2}\left(\arcsin x + \frac{3}{2} x\sqrt{1-x^2}\right), && (4.29)
\end{aligned}
$$

where $x = \omega R_0$ is the speed of the endpoints of the string.

We have thus re-discovered that the stringy meson picture coincides with the toy model discussed in section 3. In the limit when $m_q \to 0$, that is when the ends of the string are massless we recover a linear Regge trajectory when the endpoints of the string move at the speed of light $x \to 1$. More precisely, keeping the first two sub-leading terms in m_q/E we find corrections to the linear Regge trajectories of the form

$$
J = \frac{1}{\pi T_g} E^2 \left(1 + \frac{\sqrt{2}}{\sqrt{\pi}}\left(\frac{m_q}{E}\right)^{1/2} - \frac{\pi-1}{\pi}\frac{m_q}{E} + \cdots\right). \tag{4.30}
$$

It is interesting to consider the opposite limit, that is, $x \to 0$

$$
J = \frac{2m_q^{1/2}}{T_g}(E-2m_q)^{3/2} - \frac{11}{12}\frac{1}{m_q^{1/2} T_g}(E-2m_q)^{5/2} + \frac{163}{144}\frac{1}{T_g m_q^{3/2}}(E-2m_q)^{7/2}\,. \tag{4.31}
$$

The above expression is to be understood as correction to the Regge trajectory for mesons with quark masses m_q in the energy regime where $E \approx 2m_q$.

It is also possible to view the corrections in terms of the ratio of the static energy of the massive of the quarks to the energy of the static string, that is, m_q/T_gR_0. The correspondent expressions are:

$$E = \frac{2m_q\sqrt{1+q}}{q}\left(\arcsin\frac{1}{\sqrt{1+q}} + \sqrt{q}\right), \tag{4.32}$$

$$J = \frac{m_q^2(1+q)}{T_g q^2}\left(\arcsin\frac{1}{\sqrt{1+q}} + \frac{3}{2}\sqrt{\frac{q}{1+q}}\right), \tag{4.33}$$

where

$$q = \frac{m_q}{T_g R_0}\,. \tag{4.34}$$

The limit $q \to 0$ corresponds to the Regge regime whereas the limit $q \to \infty$ corresponds to the case of non-relativistic motion.

5. Regge phenomenology

It is remarkable that the Regge trajectory (J, E^2) arising from a classical macroscopic open string spinning in the non-extremal D4 geometry and ending on a probe D6 brane coincides with a known phenomenological model. Namely, the toy model reviewed in section 3. It is therefore incumbent upon us to revisit the status of this model in the phenomenology literature.

One particular observation is that the Regge trajectories of mesons are non-linear, with the non-linearity the more pronounced the heavier the masses of the constituent quarks. The trajectories for mesons composed of light quarks are essentially linear, with a universal value of the slope $\alpha' \approx 0.85\,\mathrm{GeV}^{-2}$. This class of light-quark mesons includes, for example, the trajectories of the ρ, K^*, π^0 and ω. The slope of the Regge trajectories of heavy-quark mesons varies along the trajectory, from a smaller slope for the lightest states within the trajectory (it is $\approx 0.5\,\mathrm{GeV}^{-2}$ for $c\bar{c}$), to the same universal slope for the highest states.

To exemplify the nonlinearity, we have included the $b\bar{b}$ trajectory displayed in figure 3 which is obtained from [21]. The latter paper employs a quark-antiquark effective potential derived from lattice QCD [22]. Since for $b\bar{b}$ and $c\bar{c}$ mesons the experimental data regarding resonances with spin higher than 1 are scarce, the trajectory in figure 3 is the result of a lattice analysis. Similar conclusions about the Regge trajectory can be drawn for charmonium [20] and other heavy-quark mesons [19].

Notice that the same flattening of the Regge trajectory for the lowest states within the trajectory can be inferred from plotting (J, E^2) from (4.29), as it can be seen from figure 4.

Thus, the Regge trajectory extracted from the naive toy model qualitatively captures various aspects of realistic heavy-quark meson trajectories.

The string derivation of the Regge trajectory (4.29) explains why the highest states within any meson trajectory end up with the same universal slope. The answer is beautiful and geometrical: the universal slope corresponds to the rescaled string tension of the portion of the string that spins closed to the confining wall, that is, closed to U_Λ in figure 2. This tension T_g is clearly universal and independent on the position of the probe branes, hence independent on the flavor of the quarks.

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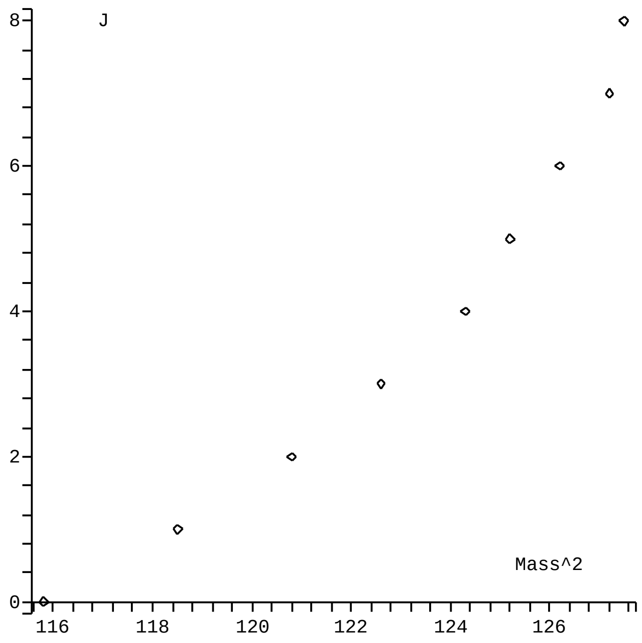

Figure 3: The bottomonium trajectory [22, 21].

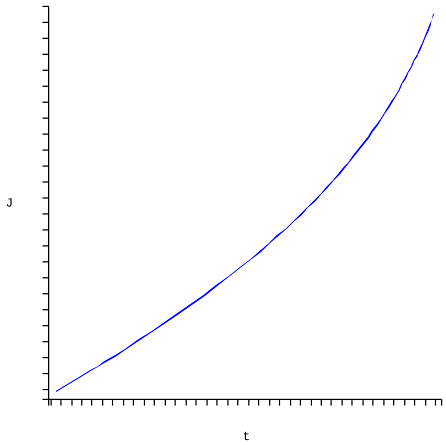

Figure 4: A parametric plot of (4.29) for $x \to 1$ shows similarity with the bottomonium trajectory.

Recently, Wilczek and collaborators [17, 23] have developed an extremely successful approach to the phenomenology of mesons and baryons. Their model includes, as an essential tool, the mass-loaded version of the Chew-Frautschi model, which is precisely the effective Regge trajectory we obtained in this paper. It would be interesting to approach other aspects of their model from our fundamental point of view and hopefully provide a *derivation* from the string model point of view of this phenomenological model.

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